Physics 177: Spring 2008 Midterm Exam

Due Monday March 17, 9 AM to Stanley Hall 608A
Work alone, but consult Jeff (jmoffitt@berkeley.edu) if you have questions.
Corrected Version: 1

Problem 1: Multivalent Polyelectrolyte

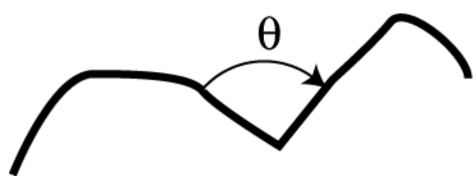
Recall that Manning argued that for a charged polymer in a monovalent salt, counterions will condense on the polymer until the condition $\xi = 1$.

(a) Derive the condition on ξ when the counter-ions are divalent. (2 points)

(b) Using this result, derive the final linear charge density (number of charges per length) of double-stranded DNA after condensation with $MgCl_2$ at 25 ˚C. (4 points)

(c) Derive the Debye length, κ^{-1}, for a polyelectrolyte with ions of arbitrary valency, Z_+ and Z_-. (Hint: Consider the linearized-Poisson-Boltzmann Equation and show that it takes the form $\nabla_r^2 \psi(r) = \kappa^2 \psi(r)$.) (4 points)

Problem 2: Worm-Like Chain Statistics



A polymer molecule of persistence length *P* and total length *L* is known to have a rigid bend at a distance *l* from one of its ends. Assuming that the bend angle is θ:

(a) Derive an expression for the mean square end-to-end distance of this molecule. (6 points)

(b) Consider the case that that $l >> P$ and $L\text{-}l >> P$. What is the mean square end-to-end distance in this case? (2 points)

Problem 3: DNA Binding Proteins

A biochemist has been studying the binding of a protein to a specific 20 bp region of double stranded DNA on a 200 bp piece of DNA and has determined that the dissociation constant for this process (K_D) at room temperature (25 ˚C) is 1.78 x 10^{-11} M (M is moles/liter). However, when the biochemist circularizes this same 200 bp piece of DNA, forming a DNA mini-circle before binding the protein, the dissociation constant changes to 3.68 x 10^{-12} M.

(a) Calculate the ΔG that corresponds to each of these dissociation reactions in kcal/mol. (4 points)
(b) How do you rationalize the difference in the ΔG for binding of the protein to linear DNA and circular DNA? (2 points)
(c) From these data, calculate the bending rigidity of DNA and its persistence length at room temperature. (4 points)

Problem 4: Entropy and Enthalpy of an Alpha Helix

A particular amino acid sequence has been found to adopt an alpha helical conformation. Experiments indicate that the melting point of the helix is 50˚C and that the enthalpy change in breaking the hydrogen bonds and disrupting the structure is ΔH^0 = **170** kJ/mol. In addition, experiments have shown that the ΔC_P for the melting of this alpha helix is negligible.

(a) What is the entropy change, ΔS^0, for this process? (3 points)
(b) How long is the helix? Assume that this entropy change is entirely the result of the degrees of freedom gained by the molecule upon melting and that each amino acid has 3 additional degrees of freedom upon melting each of which can adopt 2 conformations. (3 points)
(c) Using the length of the helix calculated above, what is the change in enthalpy per hydrogen bond in an alpha helix? (You will need to determine the maximum number of hydrogen bonds that can be formed in an alpha helix of length *N*. Neglect interactions of the side chains) (3 points)

Problem 5: Protein Binding to a Charged Surface: I

Consider a protein that can bind to a flat charged surface, such as the cell membrane or the surface of a large vesicle. Assume that this surface has a surface charge density of σ, that the protein has a net positive charge of +1*e*, where *e* is the charge of the electron, and that the protein binds at a distance r_0 from the surface of 1 Å.

(a) Derive the electrostatic potential, $\psi(r)$, at an arbitrary distance *r* away from the surface in the presence of a monovalent salt of concentration n_s. Assume that the charge density is low enough that condensation does not need to occur and the electrostatic energy of a free ion is always smaller than thermal energy. (5 points)

(b) Calculate the change in entropy upon binding if the dissociation constant (K_D) is 1 x 10^{-2} M in 50 mM NaCl at 25° C and the surface charge density is -1*e* per 2 nm^2. Assume that the enthalpy of the reaction is entirely due to electrostatic interactions. If you could not derive the potential for part a, assume that the protein is binding 1 Å away from DNA in the same salt conditions. (5 points)

(c) Provide an explanation for the sign of the change in entropy calculated above. (2 points)

Problem 6: Protein Binding to a Charged Surface: II

Consider again the protein that is binding to the charged surface in problem 5. If the only charged portions of this protein are the N-terminus ($pK_a = 8.0$), the C-terminus ($pK_a = 3.0$), and a single lysine residue ($pK_a = 10.5$), (where the pK_a values are measured when the protein is bound to the surface), then:

(a) Calculate the K_a values for these three chemical groups and the fraction of each that will be ionized at pH=7.0. (2 points)

(b) Derive an expression for the average net charge of the protein, $\langle q \rangle$, when it is bound to the surface as a function of $[H^+]$. Calculate the average charge at pH=7.0. Assume that the ionization of each group does not affect the pK_a of the other groups. (5 points)

(c) Derive an expression for the change in enthalpy upon binding as a function of $[H^+]$ in terms of the $\langle q \rangle$ calculated in part b and the potential calculated in problem 5. Neglect any changes in the pK_a values that may arise when the protein unbinds. (3 points)

(d) Find the value of pH at which the change in enthalpy upon binding is zero. (2 points)