Molecular Biophysics: Physics 177. Spring 2008.
Homework #5. Single Molecules I.

Assigned April 17. Due April 25.

1. A single DNA duplex (length L and persistence length P) is pulled between a coverslip and a bead (radius a) in an optical trap (stiffness κ) in water (viscosity η). The trapped bead can be moved relative to the coverslip to stretch the DNA. Write the corner frequency ω_c and the rms fluctuations in bead position $\langle \Delta x^2 \rangle^{1/2} << L$ as a function of distance x (where $a << x < L$) between the center of the trapped bead and the coverslip. Neglect surface effects, and assume that the relaxation time of the DNA is much faster than that of the bead in the trap. Hint: starting with the relation between DNA tension and end-to-end distance what is the stiffness of the DNA, i.e. the change in force for a change in extension.

$$F = \frac{k_B T}{P}\left[\frac{1}{4(1-x/L)^2} + \frac{x}{L} - \frac{1}{4}\right]$$

2. A ferromagnetic bead in a solution of viscosity η (radius a, magnetization M) turns in a rotating external magnetic field with strength H_o.

a) If the magnetic moment of the bead is rotated with respect to the external field by an angle θ, what is the torque on the bead?

b) What is the torque required to rotate the bead at a frequency ω against viscous drag? Ignore the magnetic torque for now.

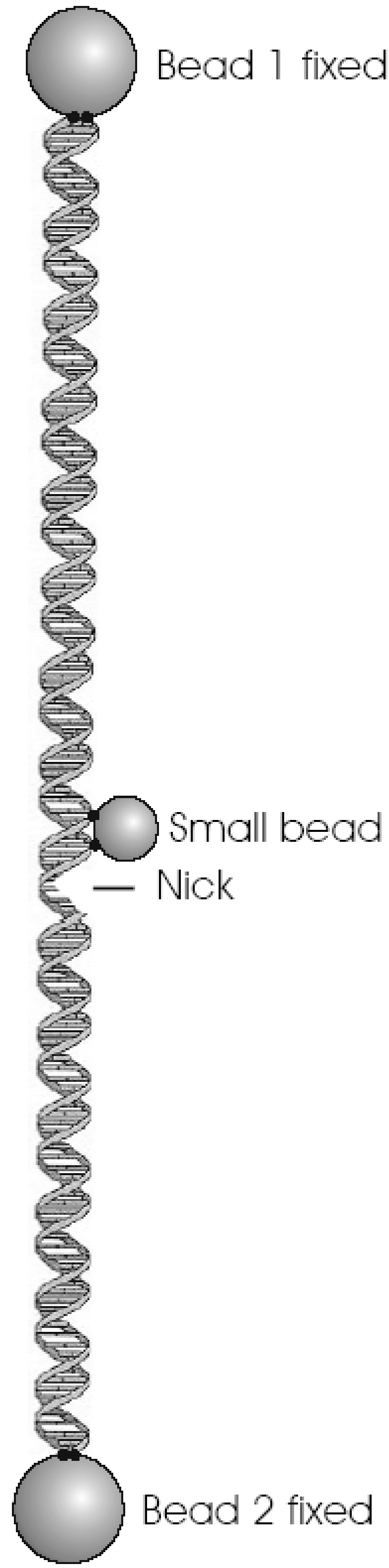
b) What is the maximum frequency at which the bead will rotate with the external field?

c) Now a single DNA duplex (length L, torsional persistence length P_T) is pulled between a similar magnetic bead and a coverslip. If the magnetic field H is oriented parallel to the coverslip, what are the rms fluctuations $\langle \Delta\theta^2 \rangle^{1/2}$ in the angle of the bead dipole orientation? Hint: use the equipartition theorem.

3. A molecule of DNA (length L) is attached by both ends via the two strands, so it is torsionally constrained. Assume that you have twisted one bead relative to the other by a total angle ϕ, and you keep the molecule under tension to prevent it from supercoiling.

a) If the constant of torsional stiffness in the molecule is C, write down the expression for the total torque stored in the molecule.

b) Now you put a tiny bead of radius R attached to both the strands and break one of the strands below the tiny bead, so that now the molecule can unwind by rotating around the single bond swivel in the unbroken strand (see figure at right). Write down an expression for the angular velocity of the small bead in terms of the viscosity η, the radius of the bead R, the constant of torsional stiffness C, and the instantaneous number of whole extra turns in the molecule, N, where $N = \phi/2\pi$.



4. Calculate the change in entropy of a single molecule of DNA of length L that has been extended in an optical tweezer between $x = \frac{1}{3}L$ and $x = \frac{2}{3}L$. Treat the molecule as a worm-like chain.